Gravity with Torsion and Matter–Antimatter Asymmetry

Nikodem Popławski

13th Australasian Conference on General Relativity and Gravitation

International Symposium on Cosmology and Particle Astrophysics

University of Canterbury, Christchurch, New Zealand

July 8, 2026

Abstract

The conservation law for the orbital plus spin angular momentum of a free Dirac particle in curved spacetime requires that the affine connection has the antisymmetric part: the torsion tensor, which extends general relativity to the Einstein–Cartan theory of gravity.

In the presence of torsion, the Dirac equation becomes a nonlinear, cubic equation in the spinor wave function.

We show that the energy eigenvalues of the corresponding Hamiltonian as functions of the momentum are different for the fermion and antifermion components of the spinor, violating charge conjugation symmetry, and also depend on the helicity.

Consequently, particles of matter and antimatter have different dispersion relations and therefore different masses.

This mass difference increases with density and becomes significant near the Cartan density, which existed in the early Universe.

Because antimatter particles were more massive than matter particles, they were also slower during pair production in the early Universe and therefore had higher cross sections for gravitational capture by primordial black holes.

This difference might have led to the matter–antimatter imbalance in the observable Universe: the missing antimatter fell into black holes.

General relativity with spin and torsion

In general relativity, the affine connection Γ_{ij}^{k} is constrained to be symmetric.

Einstein–Cartan–Sciama–Kibble (EC) gravity removes this constraint by regarding the **antisymmetric part of the connection, the torsion tensor**:

$$S^{k}{}_{ij} = \frac{1}{2}(\Gamma_{ij}^{k} - \Gamma_{ji}^{k})$$

as a variable instead of zero. The covariant derivative of the metric tensor is zero, as in GR, giving the connection $\Gamma_{ij}^{k} = \mathring{\Gamma}_{ij}^{k} + S^{k}{}_{ij} + S_{ij}{}^{k} + S_{ji}{}^{k}$, where $\mathring{\Gamma}_{ij}^{k}$ are the Christoffel symbols.

The total Lagrangian density is $-\frac{1}{2\kappa}R\sqrt{-g} + L_{\mathrm{m}}$, where R is the Ricci scalar constructed from the connection, $\kappa = 8\pi G/c^4$, and L_{m} is the Lagrangian density of matter, as in GR.

Varying the Lagrangian with respect to the contortion tensor $C_{ijk} = S_{ijk} + S_{jki} + S_{kji}$ gives the Cartan equations:

$$S^{j}{}_{ik} - S_i\delta_k^j + S_k\delta_i^j = -\frac{1}{2}\kappa s_{ik}{}^{j},$$

where $S_i = S^{k}{}_{ik}$ and $s^{ijk} = 2(\delta L_{\mathrm{m}}/\delta C_{ijk})/\sqrt{-g}$ is the spin tensor.

General relativity with spin and torsion

Varying the Lagrangian with respect to the metric tensor g_{ik} gives the Einstein equations with the Ricci tensor. They can be put into a GR form with the **combined energy–momentum tensor**:

$$G^{ik}=\kappa T^{ik}+\kappa^2\Big[-s^{ij}{}_{[l}s^{kl}{}_{j]}-\frac{1}{2}s^{ijl}s^{k}{}_{jl}+\frac{1}{4}s^{jli}s_{jl}{}^{k}+\frac{1}{8}g^{ik}(-4s^{l}{}_{j[m}s^{jm}{}_{l]}+s^{jlm}s_{jlm})\Big].$$

Dirac spinors, representing **fermions**, couple to torsion through the covariant derivative in the Lagrangian and therefore are the **source of torsion**. At macroscopic scales, Dirac fields can be averaged and described as a spin fluid:

$$s_{ij}{}^{k}=s_{ij}u^k,\quad s_{ij}u^j=0.$$

The terms in the effective energy–momentum tensor that are quadratic in the spin tensor do not vanish after averaging:

$$G^{ik}=\kappa\Big(\epsilon-\frac{1}{4}\kappa s^2\Big)u^i u^k-\kappa\Big(p-\frac{1}{4}\kappa s^2\Big)(g^{ik}-u^i u^k),$$

where

$$s^2=\frac{1}{2}s_{ij}s^{ij}>0\propto n_{\mathrm{f}}^2$$

is the averaged square of the spin density.

Spin fluid and avoidance of singularities

The Einstein–Cartan equations for a spin fluid are therefore equivalent to the Einstein equations in GR for a perfect fluid with

$$\tilde{\epsilon} = \epsilon - \alpha n_{\mathrm{f}}^2, \quad \tilde{p} = p - \alpha n_{\mathrm{f}}^2,$$

where ϵ and p are the thermodynamic energy density and pressure, n_{f} is the number density of fermions, and $\alpha = \kappa(\hbar c)^2/32$.

EC deviates from GR at densities much higher than the Cartan density $\rho_{\mathrm{C}} = m^2c^4/G\hbar^2$ (about 10^{51} kg/m^3 for electron), so it passes all tests of GR.

The negative corrections to ϵ and p act like repulsive gravity. For a FLRW universe, they are proportional to a^{-6} (like shear) where a is the scale factor. With particle production in the early universe, the power is higher than 6, thus **preventing the formation of a cosmological singularity**.

Gravitational collapse of matter in **a black hole avoids a singularity**; a nonsingular bounce forms a new, closed, growing universe on the other side of the black hole's event horizon: our Universe formed in a black hole?

Also, particle production after the bounce combined with gravitational repulsion from torsion generates a finite period of **inflation**.

Dirac equation with torsion

The covariant derivative of a spinor ψ is $\nabla_i\psi = \partial_i\psi - \Gamma_i\psi$, where the spinor connection Γ_i is given by the Fock–Ivanenko coefficients $\Gamma_i = -(1/4)\omega_{\mu\nu i}\gamma^\mu\gamma^\nu$, with the spin connection $\omega^\mu{}_{\nu i} = e^\mu_k\omega^k{}_{\nu i} = e^\mu_k\nabla_i e^k_\nu = e^\mu_k(\partial_i e^k_\nu + \Gamma^k_{ji}e^j_\nu)$ and the Dirac matrices γ^μ.

The Dirac equation for a spinor wave function representing a fermion with mass m is given by $i\hbar\gamma^\mu e^i_\mu\nabla_i\psi = mc\psi$. In EC, the covariant derivative in the Dirac equation is linear in torsion, the torsion tensor is proportional to the spin tensor because of the Cartan equations, and the spin tensor is quadratic in the spinor field and completely antisymmetric:

$$s^{\mu\nu\rho} = (1/2)i\hbar\bar{\psi}\gamma^{[\mu}\gamma^\nu\gamma^{\rho]}\psi,$$

where $\bar{\psi} = \psi^\dagger\gamma^0$ is the adjoint spinor. Consequently, the Dirac equation is **cubic** (nonlinear) in the spinor wave function (Kibble, Hehl–Datta):

$$i\hbar\gamma^\mu e^i_\mu\mathring{\nabla}_i\psi + \frac{3}{8}\kappa\hbar^2(\bar{\psi}\gamma_\mu\gamma^5\psi)\gamma^\mu\gamma^5\psi = mc\psi, \qquad (1)$$

where $\mathring{\nabla}_i$ is the covariant derivative with respect to $\mathring{\Gamma}^k_{ij}$.

At extremely high densities, torsion makes quantum mechanics nonlinear: linear superposition, wave function collapse, quantum measurement?

Dirac equation with torsion

In the locally flat coordinate system, the Dirac equation (1) can be written in the Hamiltonian form:

$$\frac{i\hbar}{c}\frac{\partial\psi}{\partial t} = -i\hbar\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}\psi + mc\beta\psi + \frac{3}{8}\kappa\hbar^2(\bar{\psi}\gamma_\mu\gamma^5\psi)\beta\gamma^\mu\gamma^5\psi, \tag{2}$$

where $\boldsymbol{\alpha}$ is the vector formed from the matrices $\alpha^\mu = \beta\gamma^\mu$ with the space indices μ, and $\beta = \gamma^0$. If the particle has four-momentum p_μ, the corresponding spinor wave function has a form of a locally plane wave proportional to $\exp(-ip_\mu x^\mu/\hbar)$, where x^μ are the spacetime coordinates. Consequently, the Dirac equation (2) becomes

$$E\psi = \mathbf{p}\cdot\boldsymbol{\alpha}c\psi + mc^2\beta\psi + k(\bar{\psi}\gamma_\mu\gamma^5\psi)\beta\gamma^\mu\gamma^5\psi = H\psi, \tag{3}$$

where $\mathbf{p}$ is the momentum, E is the energy, and $k = (3/8)\kappa\hbar^2 c$. The dimension of the wave function is thus $\mathrm{m}^{-1}\ \mathrm{s}^{-1/2}$.

Writing γ^μ in terms of the Pauli matrices σ^μ, forming the vector $\boldsymbol{\sigma}$, gives the Hamiltonian matrix H:

$$\begin{pmatrix} mc^2 I_2 + k(\psi^\dagger\boldsymbol{\alpha}\gamma^5\psi)\cdot\boldsymbol{\sigma} & \boldsymbol{\sigma}\cdot\mathbf{p}c - k\psi^\dagger\gamma^5\psi \\ \boldsymbol{\sigma}\cdot\mathbf{p}c - k\psi^\dagger\gamma^5\psi & -mc^2 I_2 + k(\psi^\dagger\boldsymbol{\alpha}\gamma^5\psi)\cdot\boldsymbol{\sigma} \end{pmatrix}, \tag{4}$$

where I_2 is the two-dimensional unit matrix.

Dirac Hamiltonian matrix with torsion

The Hamiltonian matrix is equivalent to

$$\begin{pmatrix} mc^2+kj_z & k(j_x-ij_y) & p_zc-k\rho & (p_x-ip_y)c \\ k(j_x+ij_y) & mc^2-kj_z & (p_x+ip_y)c & -p_zc-k\rho \\ p_zc-k\rho & (p_x-ip_y)c & -mc^2+kj_z & k(j_x-ij_y) \\ (p_x+ip_y)c & -p_zc-k\rho & k(j_x+ij_y) & -mc^2-kj_z \end{pmatrix}, \tag{5}$$

where $\rho = j^0 = \psi^\dagger\gamma^5\psi$ is the time component of the spin pseudovector four-current $j^i = \bar{\psi}\gamma^i\gamma^5\psi$ and its space components $\mathbf{j}$ are $j_x = \psi^\dagger\alpha_x\gamma^5\psi$ and similarly for y and z.

Diagonalizing the Hermitian matrix (5), $\det(H-\lambda I)=0$, determines the energy eigenvalues λ through a quartic equation:

$$\begin{aligned} &\lambda^4 + a\lambda^2 + b\lambda + d = 0, \\ &a = -2[(mc^2)^2 + \mathbf{p}^2c^2 + k^2\mathbf{j}^2 + k^2\rho^2], \quad b = 8k^2\rho c(\mathbf{p}\cdot\mathbf{j}), \\ &d = (mc^2)^4 + 2(mc^2)^2(\mathbf{p}^2c^2 - k^2\mathbf{j}^2 + k^2\rho^2) + \mathbf{p}^4c^4 \\ &+k^4\mathbf{j}^4 + k^4\rho^4 + 2k^2c^2\mathbf{p}^2\mathbf{j}^2 - 4k^2c^2(\mathbf{p}\cdot\mathbf{j})^2 - 2k^2c^2\mathbf{p}^2\rho^2 - 2k^4\mathbf{j}^2\rho^2. \end{aligned} \tag{6}$$

This dispersion relation determines the four possible energies of a free spinor particle in the presence of torsion.

Energy of a free spinor particle

The four roots of the quartic equation in (6) can be determined using the Ferrari method, which writes this equation in the form: $(\lambda^2 + y/2)^2 = (y - a)\lambda^2 - b\lambda + y^2/4 - d$, and chooses the quantity y such that the right side of this equation is a perfect square. This condition means that the discriminant of the quadratic equation on the right side is zero, giving a cubic equation for y:

$$b^2 - (y^2 - 4d)(y - a) = 0. \tag{7}$$

The quartic equation becomes $(\lambda^2 + y/2)^2 = (y - a)[\lambda - b/2(y - a)]^2$, so

$$\begin{aligned} \lambda^2 - (y-a)^{1/2}\lambda + y/2 + \frac{by}{4(y-a)^{1/2}} &= 0, \\ \lambda^2 + (y-a)^{1/2}\lambda + y/2 - \frac{by}{4(y-a)^{1/2}} &= 0. \end{aligned} \tag{8}$$

The cubic equation (7) reduces to a depressed cubic equation: $t^3 + pt + q = 0$, where $t = y - a/3$, $p = -(a^2/3 + 4d)$, $q = 8ad/3 - b^2 - 2a^3/27$, and has three different roots. They can be determined using the del Ferro–Tartaglia–Cardano method, which puts two quantities u and v satisfying $u+v = t, uv = -p/3$ in the depressed cubic equation, giving $u^3 + v^3 + (u+v)(p+3uv) + q = u^3 + v^3 + q = 0$. This equation becomes $u^3 - p^3/27u^3 + q = 0$, which with $w = u^3$ reduces to a quadratic equation $w^2 + qw - p^3/27 = 0$. Solving for $w \to u \to v \to t \to y$ gives y (any of the three roots suffices), which with (8) gives the eigenvalues λ.

Relation to equations of motion with torsion

Without spin and torsion, the terms with k vanish and the quartic equation (6) reduces to $[\lambda^2 - (mc^2)^2 - (\mathbf{p}c)^2]^2 = 0$, giving two double roots $\lambda = E$ (positive-energy states describing matter), where $E = [(mc^2)^2 + (\mathbf{p}c)^2]^{1/2}$, and two double roots $\lambda = -E$ (negative-energy states describing antimatter). These roots agree with the relation between the four-momentum P^i and four-velocity u^i: $P^i = mcu^i$.

In the presence of torsion, the energy as a function of the momentum is given by four different, more complicated values λ (8), which agree with the relation

$$P^i = mcu^i + (DS^{ik}/ds)u_k,$$

where S^{ik} is the four-spin, D is the covariant differential for the affine connection Γ^k_{ij}, and s is the interval. This relation is consistent with the Mathisson–Papapetrou equations of motion of particles with spin:

$$DS^{ij}/ds = P^i u^j - P^j u^i, \quad DP^i/ds = 2S_{jk}{}^{i} P^j u^k + (1/2)R_{jkl}{}^{i} S^{jk} u^l,$$

where $R^i{}_{klm}$ is the curvature tensor for the affine connection Γ^k_{ij}.

Spinor particle at rest with torsion

The character of the four energy eigenvalues λ can be determined by considering the particle in the rest frame of reference, in which its momentum is zero. Using $\mathbf{p}=0$ and $\rho=0$, which follows from $p_i j^i=0$, in (6) gives

$$a=-2[(mc^2)^2+k^2\mathbf{j}^2],\quad b=0,\quad d=[(mc^2)^2-k^2\mathbf{j}^2]^2.$$

The cubic equation (7) therefore reduces to $y^2=4d$, giving $y=2[(mc^2)^2-k^2\mathbf{j}^2]$, so $y-a=4(mc^2)^2$. The solutions (8) of the quartic equation (6) reduce to $\lambda^2\pm 2mc^2\lambda+(mc^2)^2-(k\mathbf{j})^2=0$, or $(\lambda\pm mc^2)^2=(k\mathbf{j})^2$, giving

$$\lambda=mc^2-k|\mathbf{j}|,\quad \lambda=mc^2+k|\mathbf{j}|,\quad \lambda=-mc^2-k|\mathbf{j}|,\quad \lambda=-mc^2+k|\mathbf{j}|. \quad (9)$$

Equation (6) reduces to $[\lambda^2-(mc^2-k|\mathbf{j}|)^2][\lambda^2-(mc^2+k|\mathbf{j}|)^2]=0$, giving the roots (9). A free spinor particle at rest has two positive-energy, **matter** states:

$$E=mc^2-k|\mathbf{j}|,\quad E=mc^2+k|\mathbf{j}|,$$

and two negative-energy, **antimatter** states:

$$E=-mc^2-k|\mathbf{j}|,\quad E=-mc^2+k|\mathbf{j}|.$$

In a frame of reference, in which the particle has momentum $\mathbf{p}$, these states have more complicated energies (8), depending on the helicity $\mathbf{p}\cdot\mathbf{j}$.

Matter-antimatter asymmetry from torsion

To reach a lower energy in the rest frame, a particle of **matter** prefers to be in the state with $E = mc^2 - k|\mathbf{j}|$, corresponding to the effective mass $M = E/c^2$:

$$M = m - k|\mathbf{j}|/c^2, \tag{10}$$

and a particle of antimatter prefers to be in the state with $E = -mc^2 - k|\mathbf{j}|$.

Because particles of antimatter can be regarded as positive-energy states under time inversion, the preferred energy state of a time-inversed particle of **antimatter** is $E = mc^2 + k|\mathbf{j}|$, corresponding to the effective mass

$$M = m + k|\mathbf{j}|/c^2. \tag{11}$$

Therefore, particles of matter and antimatter in the presence of torsion have different effective masses, which are also different from the spinor mass m that determines the dispersion relation in the absence of torsion.

On the average, **particles of antimatter in the presence of torsion have larger masses than the corresponding particles of matter**. Because a spinor has two positive-energy and two negative-energy states, the relations (10) give an inequality $k|\mathbf{j}| < mc^2$, determining an upper limit on $|\mathbf{j}|$ and thus on the mass density in the Universe.

Matter–antimatter asymmetry from torsion

The energy of a free Dirac particle in the presence of torsion therefore depends on whether it is a matter particle or an antimatter particle, violating C-symmetry.

The energy also depends on helicity, violating CP-symmetry.

These violations constitute one of the three Sakharov necessary conditions for **baryogenesis**.

At densities much smaller than the Cartan density, $k|\mathbf{j}| \ll mc^2$, so the effects of torsion are negligible and so are these violations. At these densities, the masses of corresponding matter and antimatter particles are almost equal, in accordance with the results of experimental measurements of the masses of the proton and antiproton.

The matter–antimatter mass asymmetry, and the corresponding helicity asymmetry, might be responsible for the observed spin asymmetry in the proton, and could possibly be measured directly in experiments probing smaller distances and higher densities.

Torsion-generated asymmetry in gravitational capture by black holes

The effective cross-section for gravitational capture of a particle with mass m, moving with velocity v_∞ at spatial infinity, by a Schwarzschild black hole with a mass μ can be determined from the effective potential energy:

$$U(r) = mc^2 \left[\left(1 - \frac{r_g}{r}\right) \left(1 + \frac{M^2}{m^2 c^2 r^2}\right) \right]^{1/2},$$

where $r_g = 2G\mu/c^2$ is the Schwarzschild radius of the black hole and M is the angular momentum of the particle. In the radial equation of motion, $(1 - r_g/r)^{-1} dr/c\,dt = (1/E)(E^2 - U^2)^{1/2}$, the condition $E \geq U$ gives the range of admissible motion of the particle. Its conserved energy is $E = mc^2\gamma$, where $\gamma = (1 - v_\infty^2/c^2)^{-1/2}$, and its conserved angular momentum is $M = m\rho v_\infty \gamma$, where ρ is the impact parameter.

The particle falls into the black hole if $E > U_{\max}$, where the maximum value $U_{\max}$ of the effective potential energy is given by $dU/dr = 0$ and $d^2U/dr^2 < 0$. These conditions give $\rho < \rho_{\max}$, from which the capture cross-section is $\sigma = \pi \rho_{\max}^2$.

Torsion-generated asymmetry in gravitational capture by black holes

The effective cross-section for gravitational capture by a Schwarzschild black hole is

$$\sigma = \frac{1}{8}\pi r_g^2\Big[-(\alpha^2-18\alpha-27)+\sqrt{\alpha^4-28\alpha^3+270\alpha^2+972\alpha+729}\Big],$$

where $\alpha=(\gamma^2-1)^{-1}$. For an ultrarelativistic particle, $\alpha\ll 1$ gives

$$\sigma \approx \frac{27}{4}\pi r_g^2\left(1+\frac{2}{3\gamma^2}\right). \tag{12}$$

For light, this formula gives $\sigma = (27/4)\pi r_g^2$. For a nonrelativistic particle, $\alpha\to\infty$ gives $\sigma\to 4\pi r_g^2 c^2/v_\infty^2$.

Gravitational capture asymmetry

In the early Universe, strong and time-varying gravitational fields caused intense pair production of fermions. In the rest frame of reference of a matter–antimatter fermion pair, the conservation of momentum gives $M_q v_q \gamma_q = M_{\bar{q}} v_{\bar{q}} \gamma_{\bar{q}}$, where q denotes matter and $\bar{q}$ denotes antimatter. For ultrarelativistic particles, their velocities were $v_q \approx v_{\bar{q}} \approx c$, giving

$$\gamma_{\bar{q}}/\gamma_q \approx M_q/M_{\bar{q}}. \tag{13}$$

In the presence of torsion, the effective masses are $M_- = m - k|\mathbf{j}|/c^2$ and $M_+ = m + k|\mathbf{j}|/c^2$, following (10) and (11). Particles of matter preferred to be in the state with M_- and particles of antimatter preferred to be in the state with M_+. The energy difference between the two states is $\Delta E = 2k|\mathbf{j}|$. In thermodynamical equilibrium at temperature T, the ratios of the numbers of particles in these states are given by the Boltzmann distribution:

$$N_{q+}/N_{q-} = N_{\bar{q}-}/N_{\bar{q}+} = \exp\left(-2k|\mathbf{j}|/k_{\mathrm{B}}T\right). \tag{14}$$

Subscripts $-$ and $+$ denote the states with M_- and M_+. These numbers also satisfy $N_{q+} + N_{q-} = N_{\bar{q}-} + N_{\bar{q}+} = N$, where N is the number of all matter fermions, equal to that of all antimatter fermions in pair production.

Gravitational capture asymmetry

The value of $|\mathbf{j}|$ can be estimated by normalization of the spinor wave function. Following (3), the dimension of the volume integral of $\bar{\psi}\psi$ is the same as that of c. The volume of a closed Universe is $2\pi^2 a^3$, where a is the scale factor. The volume occupied by the wave function of each fermion is thus approximately $2\pi^2 a^3/2N$, equal to the inverse of the number density of all fermions $n = 2N/2\pi^2 a^3$, so $|\mathbf{j}| \sim \bar{\psi}\psi$ is given by $\bar{\psi}\psi(2\pi^2 a^3/2N) = c$:

$$|\mathbf{j}| \approx nc. \tag{15}$$

The relations (14) and (15) determine the numbers of fermion particles:

$$N_{q+} = N_{\bar{q}-} = \frac{N}{1 + e^{2knc/k_{\mathrm{B}}T}}, N_{q-} = N_{\bar{q}+} = \frac{Ne^{2knc/k_{\mathrm{B}}T}}{1 + e^{2knc/k_{\mathrm{B}}T}}. \tag{16}$$

The mean effective cross-sections for gravitational capture of fermions are given by (12) and (16):

$$\begin{aligned}
\sigma_q &= \frac{27}{4N}\pi r_g^2 \left[\left(1 + \frac{2}{3\gamma_-^2}\right) N_{q-} + \left(1 + \frac{2}{3\gamma_+^2}\right) N_{q+}\right], \\
\sigma_{\bar{q}} &= \frac{27}{4N}\pi r_g^2 \left[\left(1 + \frac{2}{3\gamma_-^2}\right) N_{\bar{q}-} + \left(1 + \frac{2}{3\gamma_+^2}\right) N_{\bar{q}+}\right].
\end{aligned} \tag{17}$$

Gravitational capture asymmetry

The relative difference of the cross-sections for matter and antimatter particles is therefore equal to

$$\frac{\Delta\sigma}{\sigma_q} = \frac{\sigma_{\bar{q}} - \sigma_q}{\sigma_q} = \frac{2}{3}\frac{(e^{2knc/k_{\mathrm{B}}T} - 1)(1/\gamma_+^2 - 1/\gamma_-^2)}{(1 + 2/3\gamma_-^2)e^{2knc/k_{\mathrm{B}}T} + (1 + 2/3\gamma_+^2)},$$

which for ultrarelativistic particles ($\gamma \gg 1$) reduces to

$$\frac{\Delta\sigma}{\sigma_q} \approx \frac{2}{3\gamma_-^2}\tanh\left(\frac{knc}{k_{\mathrm{B}}T}\right)\left(\frac{\gamma_-^2}{\gamma_+^2} - 1\right). \tag{18}$$

This relation can be simplified, using $\gamma_-/\gamma_+ \approx M_+/M_-$ in the last term, which follows from (13) and $M_+ \approx M_-$:

$$\gamma_-^2/\gamma_+^2 - 1 \approx M_+^2/M_-^2 - 1 \approx 2\Delta M/M_-,$$

where $\Delta M = M_+ - M_- = 2k|\mathbf{j}|/c^2 \approx 2kn/c$ is the effective mass difference and $M_- \approx m$. The mean value of γ_-^2 in (18) can be derived from the relativistic equipartition theorem, giving $mc^2\gamma = 3k_{\mathrm{B}}T$, so $\gamma_-^2 \approx 9(k_{\mathrm{B}}T/mc^2)^2$, so

$$\frac{\Delta\sigma}{\sigma_q} \approx \frac{8kn}{3m\gamma_-^2 c}\tanh\left(\frac{knc}{k_{\mathrm{B}}T}\right) \approx \frac{8kmnc^3}{27(k_{\mathrm{B}}T)^2}\tanh\left(\frac{knc}{k_{\mathrm{B}}T}\right). \tag{19}$$

Gravitational capture asymmetry

The equilibrium fermion number density is

$$n = (\zeta(3)/\pi^2)(3g/4)(k_{\mathrm{B}}T)^3/(\hbar c)^3, \tag{20}$$

where $g = 90$ is the number of thermal degrees of freedom for all known elementary fermions. Using the Planck mass m_{P} and temperature T_{P} gives the formula for the relative cross-section difference in terms of the original mass m and the temperature T of the Universe:

$$\frac{\Delta\sigma}{\sigma_q} \approx \frac{2\zeta(3)g}{3\pi}\frac{m}{m_{\mathrm{P}}}\frac{T}{T_{\mathrm{P}}}\tanh\left(\frac{9\zeta(3)g}{4\pi}\frac{T^2}{T_{\mathrm{P}}^2}\right) \approx \frac{3\zeta^2(3)g^2}{2\pi}\frac{m}{m_{\mathrm{P}}}\frac{T^3}{T_{\mathrm{P}}^3}, \tag{21}$$

using $\tanh(x) \approx x$ for $T \ll T_{\mathrm{P}}$. Even for $T \sim T_{\mathrm{P}}$, this quantity is very low because $m \ll m_{\mathrm{P}}$.

The mass difference between a fermion and the corresponding antifermion increases with density and becomes significant near the Cartan density, which existed in the early Universe. Because **antimatter particles were more massive** than matter particles during pair production in the early Universe, they were **slower**. Because the cross-section for capture of a particle by a black hole increases as its speed decreases, the **antimatter particles had a larger cross section** than the matter particles.

Baryogenesis from black holes and torsion

If the early Universe had many **primordial black holes**, more antimatter particles than matter particles fell into them, producing the matter–antimatter imbalance: the missing antimatter fell into black holes and the remaining antimatter annihilated with matter, producing photons and a surplus of matter.

This mechanism does not violate the conservation of baryon number and does not need physics beyond the Standard Model.

It satisfies the three Sakharov conditions for baryogenesis. The apparent baryon-number violation follows from antimatter falling into black holes at higher rates than matter. C-symmetry and CP-symmetry violations follow from the mass difference between matter and antimatter and its dependence on helicity. Interactions out of thermal equilibrium follow from one-way motion through the event horizon of a black hole.

Baryogenesis from black holes and torsion

Because almost all antimatter in the Universe, which was not captured by primordial black holes, annihilated with matter and produced photons, (21) is the baryon-to-photon ratio η in the Universe. This quantity is very low, which explains the enormous abundance of photons over matter. It is much lower than η measured at about 6.1×10^{-10}.

However, torsion from spin of fermions averaged as a spin fluid generates a negative correction to the energy density, which acts like gravitational repulsion. Pair production in the early Universe, caused by strong time-varying gravitational fields, increased this repulsion, giving a finite period of exponential expansion of the Universe and thus the simplest explanation of cosmic inflation.

Pair production violated thermal equilibrium, so the relation $n \sim T^3$ (20) was not satisfied and n in (19) increased by many orders of magnitude. Also, the fraction of particles that fell into black holes is proportional to the fraction of space occupied by them during pair production, decreasing (21). These two factors must be included before comparing (21) to η.

Acknowledgments

Gabriel Unger, Jordan Cubero, Francisco Guedes, Michael Del Grosso
Dr. Shantanu Desai

References

Gravity with torsion:

Kibble, JMP 2, 212 (1961); Sciama, RMP 36, 463 (1964)
Hehl & Datta, JMP 12, 1334 (1971); Hehl, von der Heyde & Kerlick, PRD 10, 1066 (1974)
Hehl, von der Heyde, Kerlick & Nester, RMP 48, 393 (1976)
NP, *Classical physics: spacetime and fields*, arXiv:0911.0334 (2024)

Universe in a black hole with spin and torsion:

NP, PLB 687, 110 (2010); PLB 694, 181 (2010); GRG 44, 1007 (2012); PRD 85, 107502 (2012);
Desai & NP, PLB 755, 183 (2016); Cubero & NP, CQG 37, 025011 (2020)

Big bounce and inflation from torsion:

NP, ApJ 832, 96 (2016); Unger & NP, ApJ 870, 78 (2019); NP, JETP 132, 374 (2021);
NP, Regular Black Holes, p. 445 (Springer, 2023)

Matter–antimatter asymmetry from torsion:

Kerlick, PRD 12, 3004 (1975); NP, PRD 83, 084033 (2011); arXiv:2101.04212 (2026)

Torsion in quantum mechanics and quantum field theory – no infinities:

NP, PLB 690, 73 (2010); FP 50, 900 (2020); Guedes & NP, CQG 41, 065011 (2024);
Del Grosso & NP, CQG 41, 225001 (2024)

Summary

The conservation law for the orbital plus spin angular momentum of a free Dirac particle in curved spacetime requires that the affine connection has the antisymmetric part: the torsion tensor, which extends general relativity to the Einstein–Cartan theory of gravity.

In the presence of torsion, the Dirac equation becomes a nonlinear, cubic equation in the spinor wave function.

We show that the energy eigenvalues of the corresponding Hamiltonian as functions of the momentum are different for the fermion and antifermion components of the spinor, violating charge conjugation symmetry, and also depend on the helicity.

Consequently, particles of matter and antimatter have different dispersion relations and therefore different masses.

This mass difference increases with density and becomes significant near the Cartan density, which existed in the early Universe.

Because antimatter particles were more massive than matter particles, they were also slower during pair production in the early Universe and therefore had higher cross sections for gravitational capture by primordial black holes.

This difference might have led to the matter–antimatter imbalance in the observable Universe: the missing antimatter fell into black holes.