Big Bounce and Inflation from Spin and Torsion

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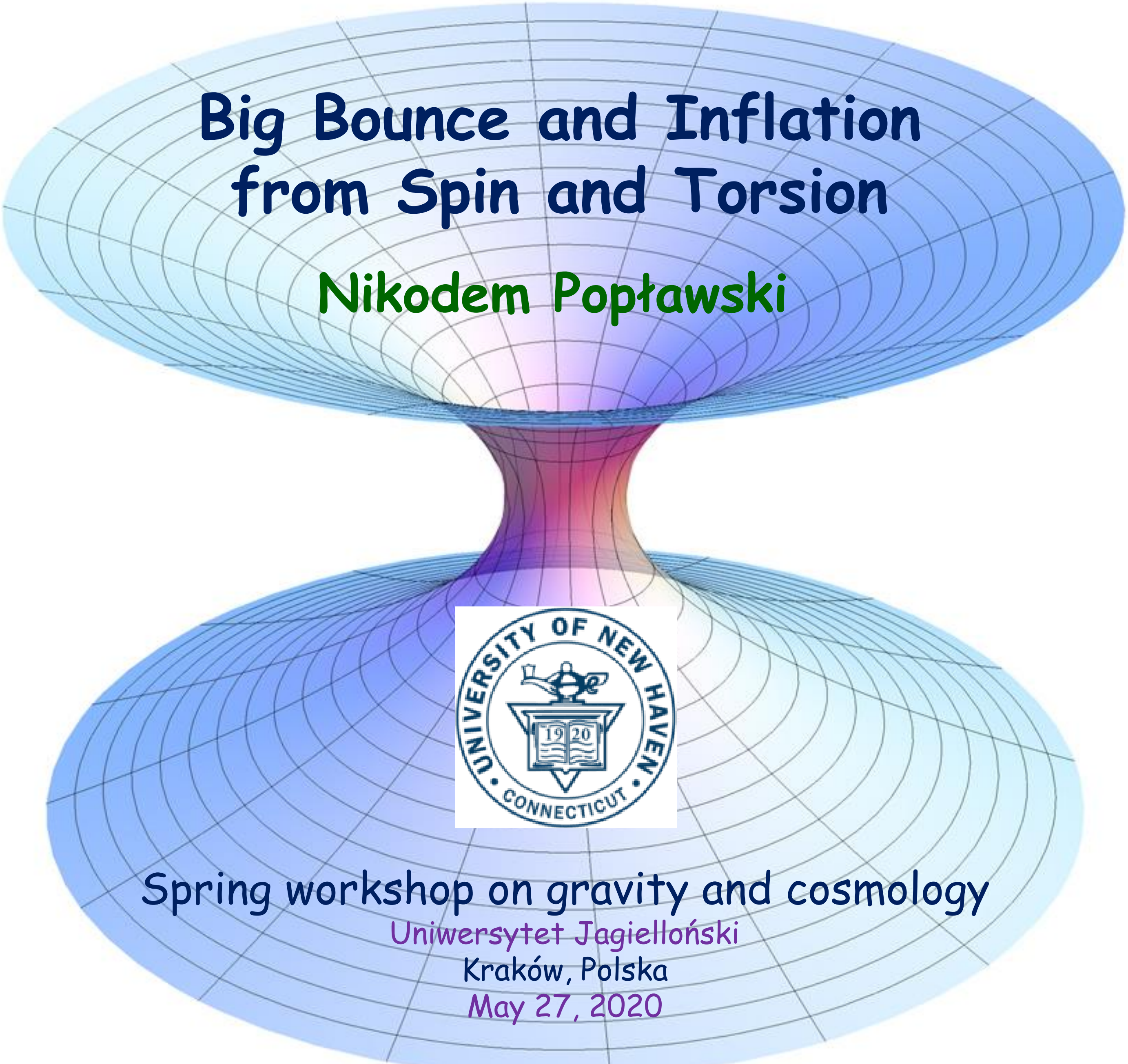

Spring workshop on gravity and cosmology

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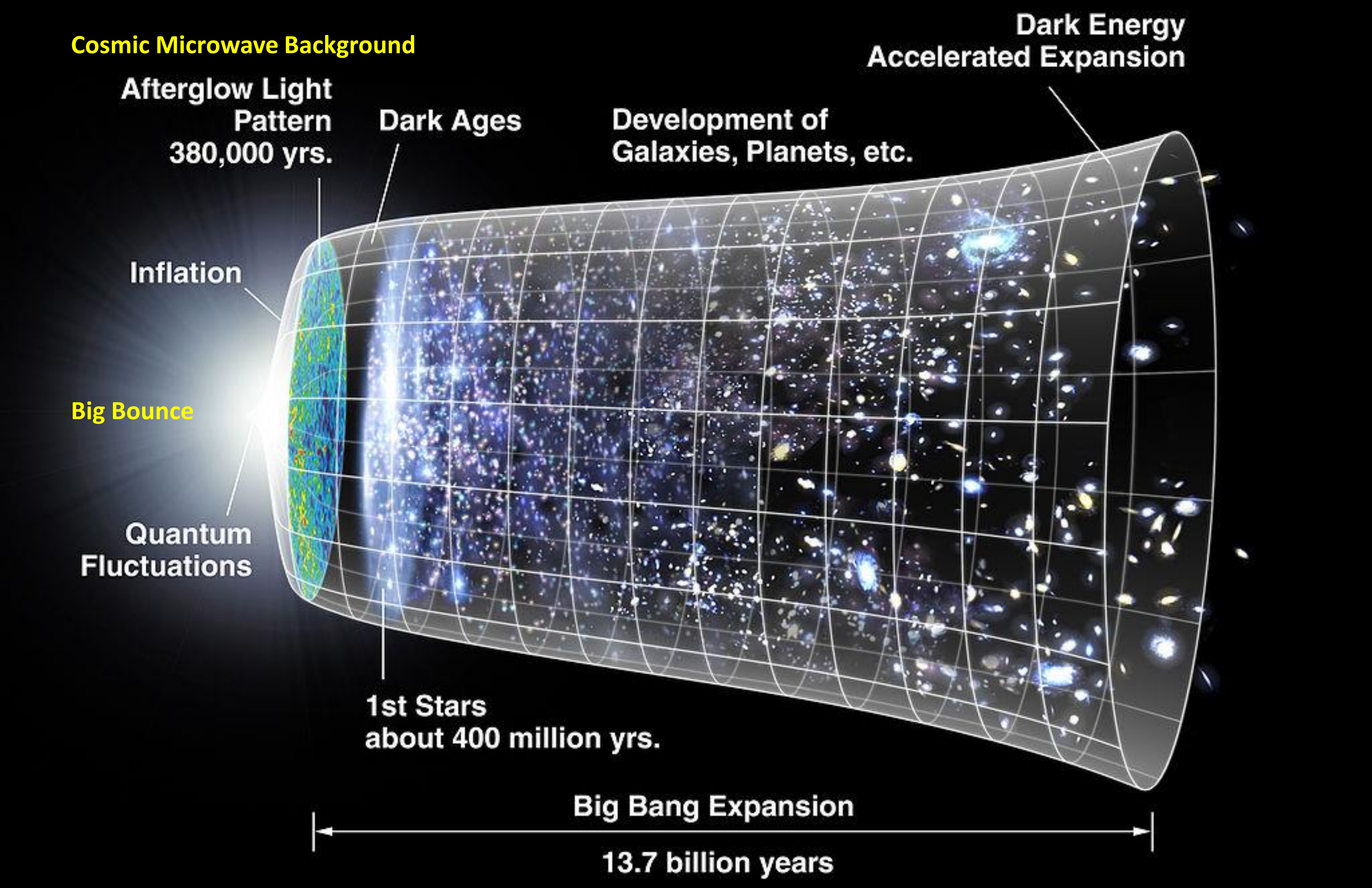

Cosmic Microwave Background
Afterglow Light Pattern 380,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Dark Energy Accelerated Expansion
Inflation
Big Bounce
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.7 billion years

Problems of General Relativity

Cosmology is based on General Relativity, which describes gravity as curvature of spacetime.

- Singularities: points with infinite density of matter.
- Incompatible with quantum mechanics. We need quantum gravity. It may resolve the singularity problem.
- Field equations contain the conservation of orbital angular momentum, contradicting Dirac equation which gives the conservation of total angular momentum (orbital + spin) and allows spin-orbit exchange in QM.

Simplest extension of GR to include QM spin:
Einstein-Cartan theory. It may resolve the singularity problem.

Problems of Big-Bang cosmology & inflation

- Big-Bang singularity.
- What caused the Big Bang? What existed before?
- Inflation (exponential expansion of the early Universe) solves the flatness and horizon problems, and predicts the observed spectrum of CMB perturbations. What caused inflation? (hypothetical scalar fields are usually used)
- How did inflation end? (no eternal inflation)

Einstein-Cartan theory replaces the Big Bang by a non-singular **Big Bounce**. The dynamics after the bounce may explain the flatness & horizon problems. NP, PLB 694, 181 (2010).

Closed Universe

If the Universe is closed, it is analogous to the 2-dimensional surface of a 3-dimensional sphere. The Universe would be mathematically the 3-dimensional hypersurface of a 4-dimensional hypersphere.

The 3-dimensional space in which the balloon expands is not analogous to any higher dimensional space. Points off the surface of the balloon are not in the Universe in this analogy.

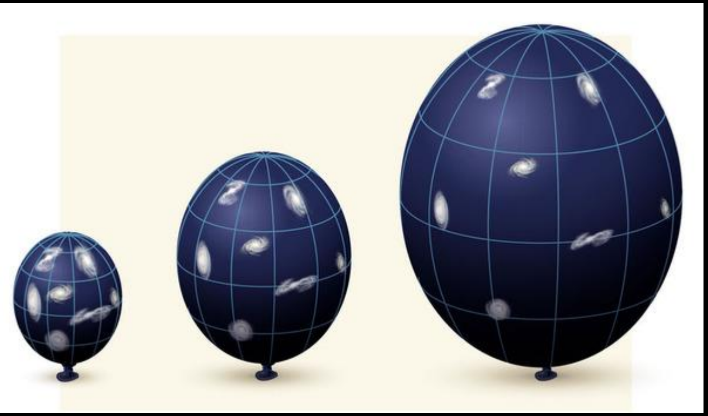


Image credit: One-Minute Astronomer

The balloon radius = scale factor a.

The Universe expands (Hubble law).

The Universe may be finite (closed) or infinite (flat or open).

Einstein-Cartan-Sciama-Kibble gravity

- The torsion tensor is a variable in addition to the metric.
- The torsion tensor is the antisymmetric part of the affine connection.

$$S^k{}_{ij} = \Gamma^{\ k}_{[i\,j]}$$

- The covariant derivative of the metric vanishes (as in GR), and the connection has a purely metric part and a torsion part.
- The Lagrangian density is proportional to the Ricci curvature scalar R (as in GR), constructed from the connection.
  The curvature tensor can be decomposed into a purely metric part and a part that contains the torsion and its derivatives.

arXiv.org > gr-qc > arXiv:0911.0334

Einstein-Cartan-Sciama-Kibble gravity

- Variation of the total action for gravity and matter with respect to the torsion gives the Cartan field equations:

$$S_{jik} - S_i g_{jk} + S_k g_{ji} = -\frac{1}{2}\kappa s_{ikj}$$

Torsion is proportional to **spin** density of fermions. ECSK differs significantly from GR at densities > 10^{45} kg/m^3. In vacuum, it reduces to GR. ECSK passes all tests that GR does.

- Variation with respect to the metric gives the Einstein equations:

Curvature is proportional to **energy and momentum** density. Using curvature decomposition, they can be written as GR with the energy-momentum tensor with terms quadratic in spin density.

$$G^{ik} = \kappa T^{ik} + \frac{1}{2}\kappa^2\left(s^{ij}{}_j s^{kl}{}_l - s^{ij}{}_l s^{kl}{}_j - s^{ijl} s^k{}_{jl} + \frac{1}{2}s^{jli} s_{jl}{}^k + \frac{1}{4}g^{ik}(2s_j{}^l{}_m s^{jm}{}_l - 2s_j{}^l{}_l s^{jm}{}_m + s^{jlm} s_{jlm})\right)$$

Universe with spin fluid

Dirac particles can be averaged macroscopically as a spin fluid.

$s^{\mu\nu\rho} = s^{\mu\nu}u^{\rho}$ $s^{\mu\nu}u_{\nu} = 0$ $s^2 = s^{\mu\nu}s_{\mu\nu}/2$

Effective energy density and pressure are $\varepsilon - \kappa s^2/4$ and $p - \kappa s^2/4$, and violate energy conditions at high s^2, evading singularity theorems.

We assume the Universe is closed, homogeneous, and isotropic (FLRW). Einstein-Cartan equations become Friedmann equations for scale factor a.

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa\left(\epsilon - \frac{1}{4}\kappa s^2\right)a^2$$

$$\frac{\dot{a}^2 + 2a\ddot{a}}{c^2} + 1 = -\kappa\left(p - \frac{1}{4}\kappa s^2\right)a^2$$

$$s^2 = \frac{1}{8}(\hbar c n)^2$$

Spin and torsion modify the energy density and pressure with a negative term proportional to the square of the fermion number density n, acting like repulsive gravity (Hehl, Kopczyński, Trautman, Kuchowicz).

Universe with spin fluid

For relativistic matter, Friedmann equations can be written in terms of temperature: $\varepsilon \approx 3p \sim T^4$, $n \sim T^3$.

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa(h_\star T^4 - \alpha h_{nf}^2 T^6)a^2$$

$$\frac{\dot{a}}{a} + \frac{\dot{T}}{T} = 0 \qquad \alpha = \kappa(\hbar c)^2/32$$

$$x = \frac{T}{T_{\rm cr}} \quad y = \frac{a}{a_{\rm cr}} \quad \tau = \frac{ct}{a_{\rm cr}}$$

$$T_{\rm cr} = \left(\frac{2h_\star}{3\alpha h_{nf}^2}\right)^{1/2} \quad a_{\rm cr} = \frac{9\hbar c}{8\sqrt{2}}\left(\frac{\alpha h_{nf}^4}{h_\star^3}\right)^{1/2}$$

Use nondimensional quantities, where critical values are on Planck order & depend on # of particle species

$$\dot{y}^2 + 1 = (3x^4 - 2x^6)y^2$$

$$xy = C > 0$$

NP, ApJ 832, 96 (2016); G. Unger & NP, ApJ 870, 78 (2019)

Generating nonsingular bounce

$$\dot{y}^2 + 1 = \frac{3C^4}{y^2} - \frac{2C^6}{y^4}$$

$$y_\pm^2 = 3C^4\left[\frac{1 \pm \sqrt{1 - \frac{8}{9C^2}}}{2}\right]$$

Turning points ($\dot{y} = 0$) for the closed Universe with torsion are positive – **no cosmological singularity!**

- 2 points if $C > (8/9)^{1/2}$ (absolute $y_{min} = 1$ for $C = 1$)
- 1 point if $C = (8/9)^{1/2}$ -> stationary Universe
- 0 points if $C < (8/9)^{1/2}$ -> Universe cannot exist (form)

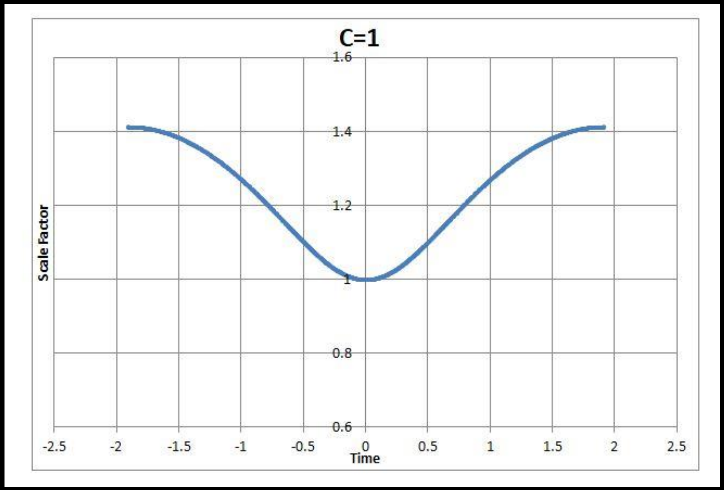
C	$y^2_{\rm min}$	$y^2_{\rm max}$	$x^2_{\rm max}$	$x^2_{\rm min}$
$\sqrt{8/9}$	$\frac{32}{27}$	$\frac{32}{27}$	$\frac{3}{4}$	$\frac{3}{4}$
1	1	2	1	$\frac{1}{2}$
$\gg 1$	$\frac{2C^2}{3}$	$3C^4$	$\frac{3}{2}$	$\frac{1}{3C^2}$

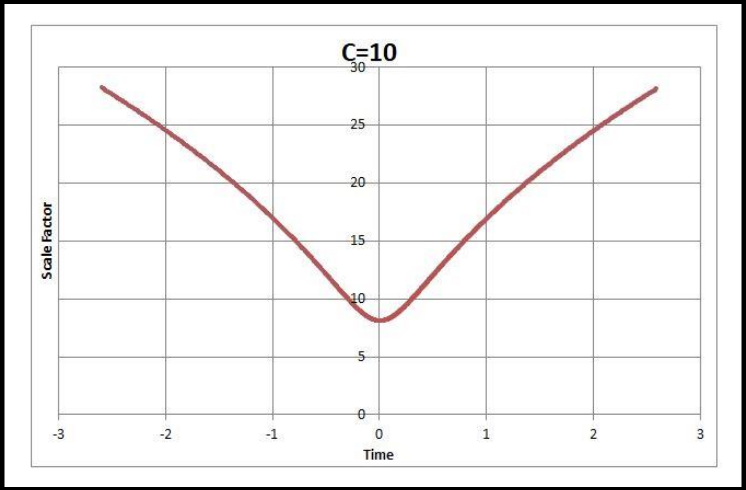
$C = aT / a_{cr}T_{cr}$

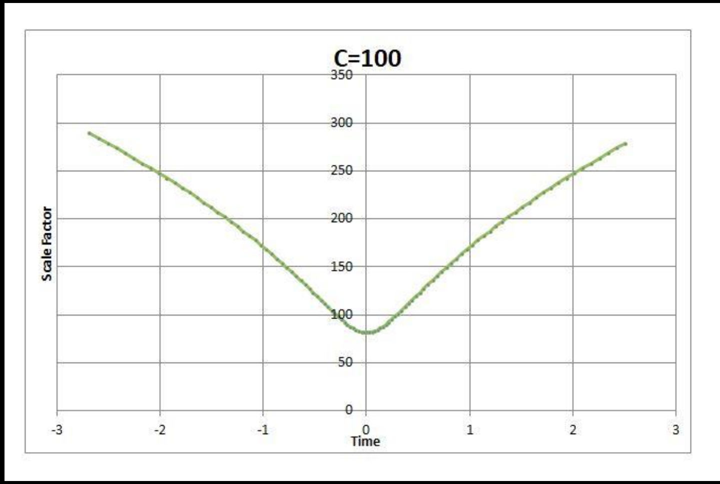
Closed universe – product aT has a lower limit.

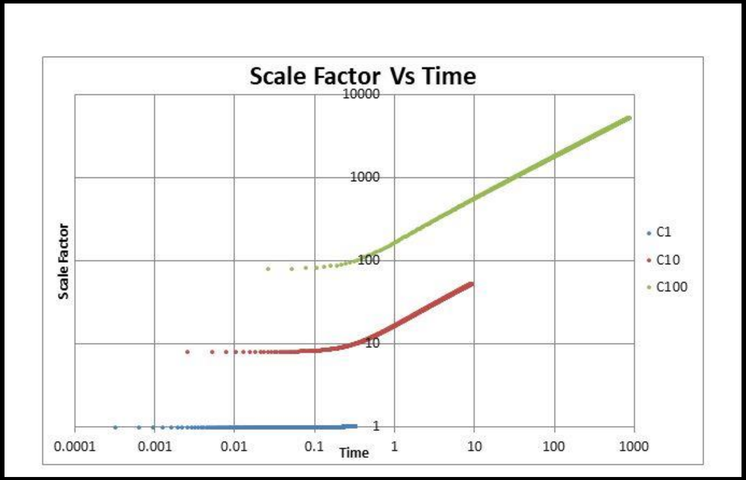
Flat & open – singularity is also avoided, but no constraint on C.

Generating nonsingular bounce





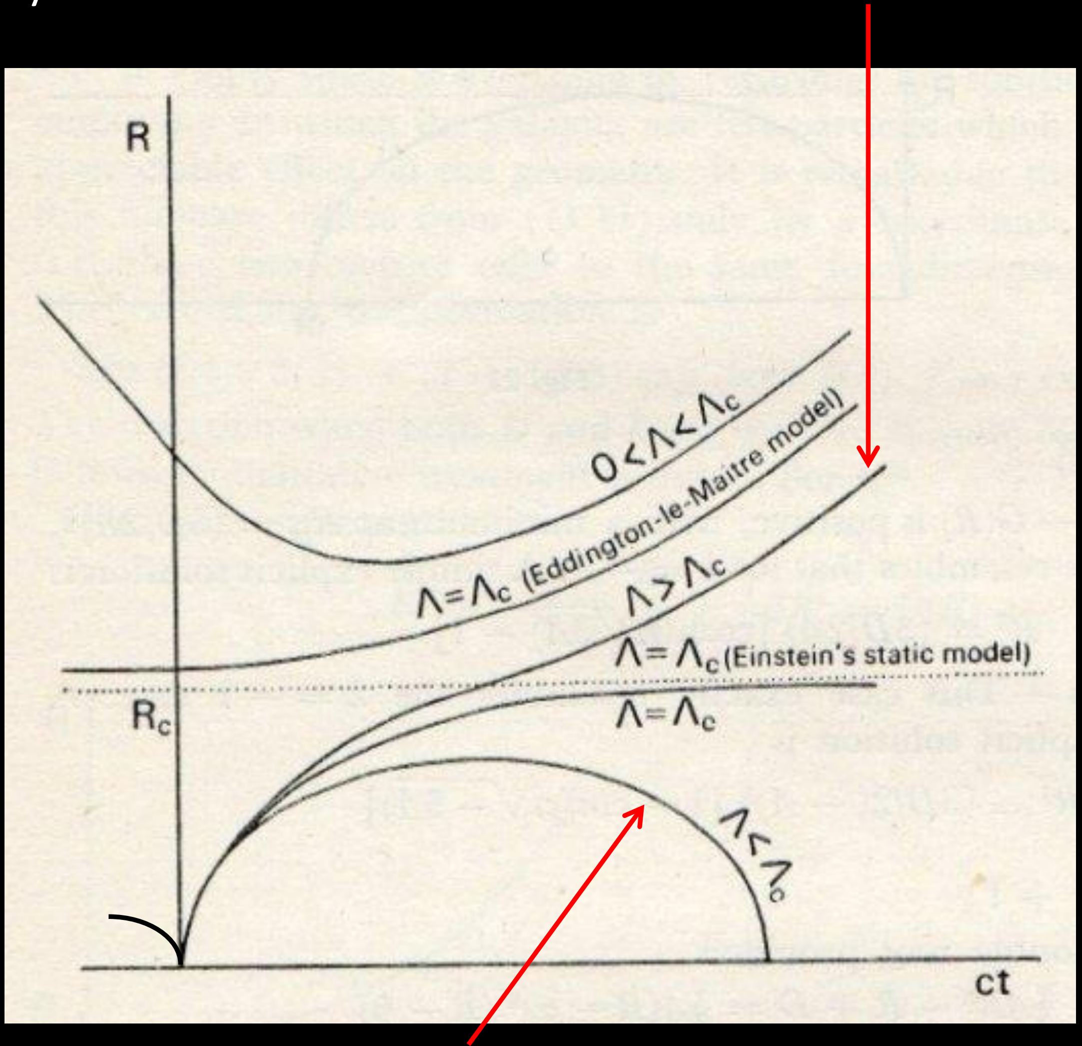




Proposal: the Universe began at $a \sim a_{cr}$ and $T \sim T_{cr}$ with $C \sim 1$.

Quantum matter production: C increased to the current $> 10^{30}$.

If quantum effects in the gravitational field near a bounce produce enough matter, then the closed Universe can reach a size at which dark energy becomes dominant and expands to infinity.



Otherwise, the Universe contracts to another bounce (with larger scale factor) at which it produces more matter, and expands again.

(Image: Lord, *Tensors, Relativity, and Cosmology*.)

Matter production causing inflation

Near a bounce, Parker-Starobinsky-Zel'dovich particle production enters through a term ~ H^4, with β as a production parameter.

$$\frac{\dot{a}}{a}\Big[1-\frac{3\beta}{c^3 h_{n1} T^3}\Big(\frac{\dot{a}}{a}\Big)^3\Big] = -\frac{\dot{T}}{T}$$

NP, ApJ 832, 96 (2016)

To avoid eternal inflation: the β term must be < 1.
This gives an upper limit on: $\beta < \beta_{cr} \approx 1/929$.

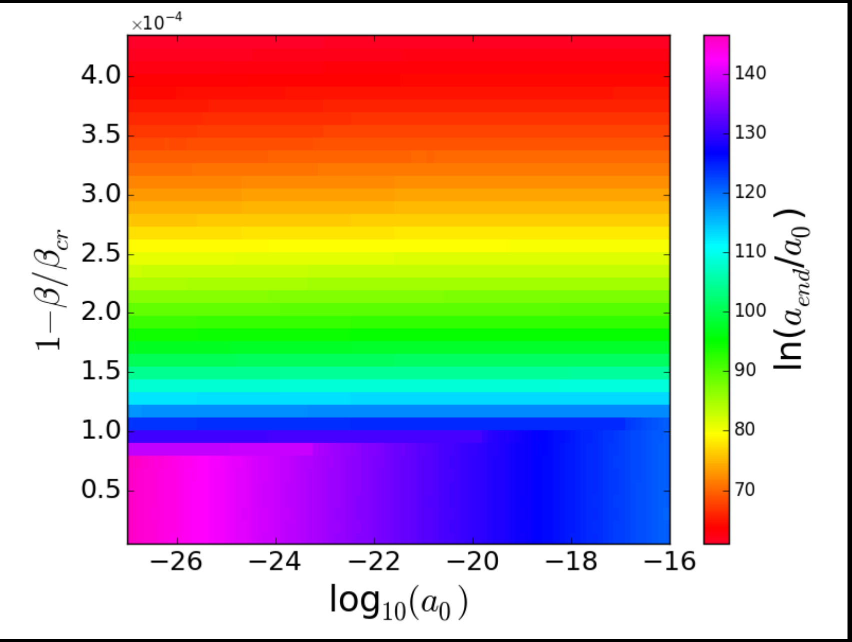
For $\beta \approx \beta_{cr}$ and during an expansion phase, when $H = \dot{a}/a$ reaches a maximum, the β term is slightly lesser than 1 and:

T ~ const, H ~ const, ε ~ const.

Exponential expansion and mass increase last about t_{Planck}, then H and T decrease. Torsion becomes weak, inflation ends, and radiation dominated era begins. No scalar fields needed.

- The numbers of e-folds and bounces (until the Universe reaches the radiation-matter equality) depend on the particle production but are not too sensitive to the initial scale factor.
- The Big Bang was the last bounce (Big Bounce).

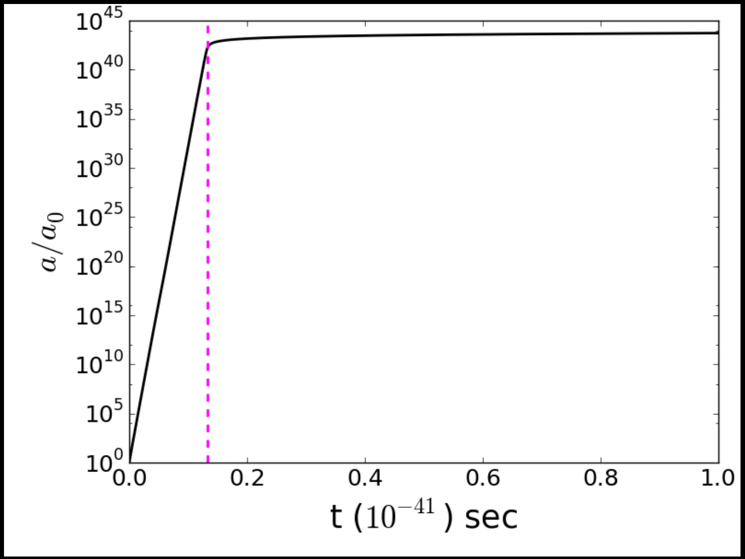
β/β_{cr}	Number of bounces
0.996	1
0.984	2
0.965	3
0.914	5
0.757	10

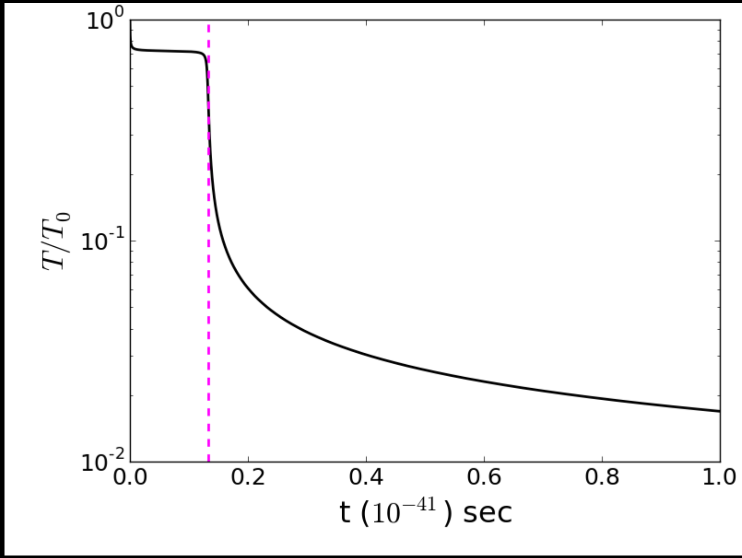


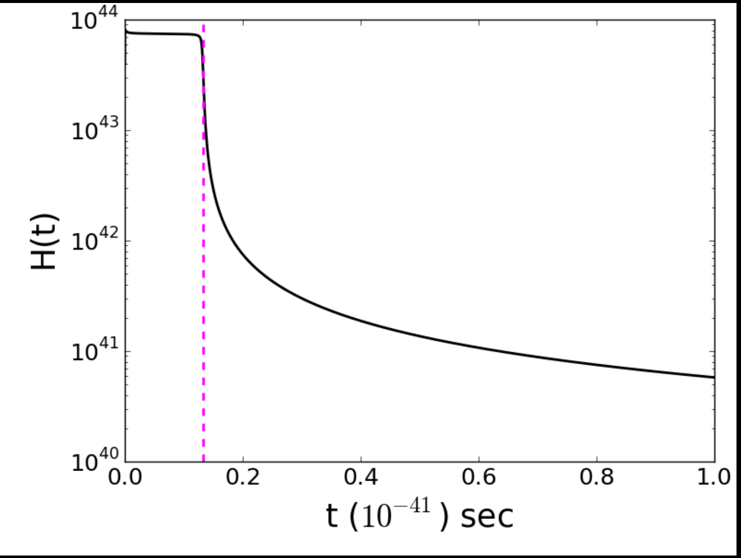
S. Desai & NP, PLB 755, 183 (2016)

Dynamics of the
very early Universe with:

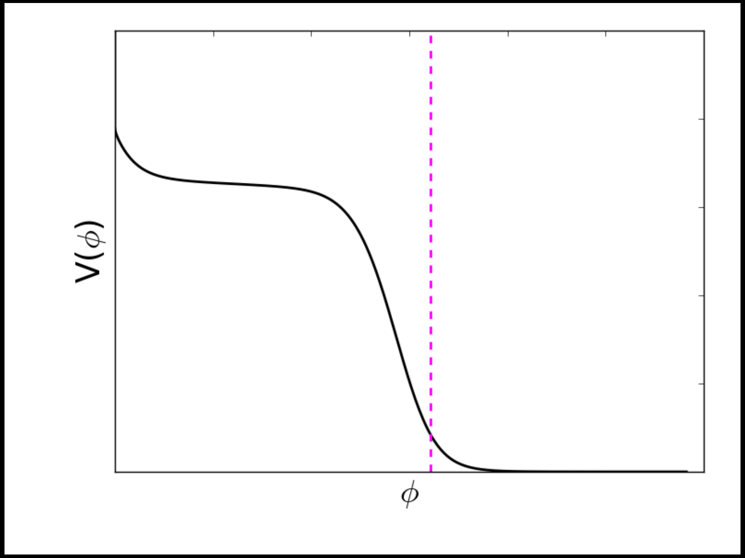
$\beta/\beta_{cr} = 0.9998$
Initial scale factor $a_0 = 10^{-27}$ m







It is possible to find a scalar field potential which generates the same time dependence of the scale factor.

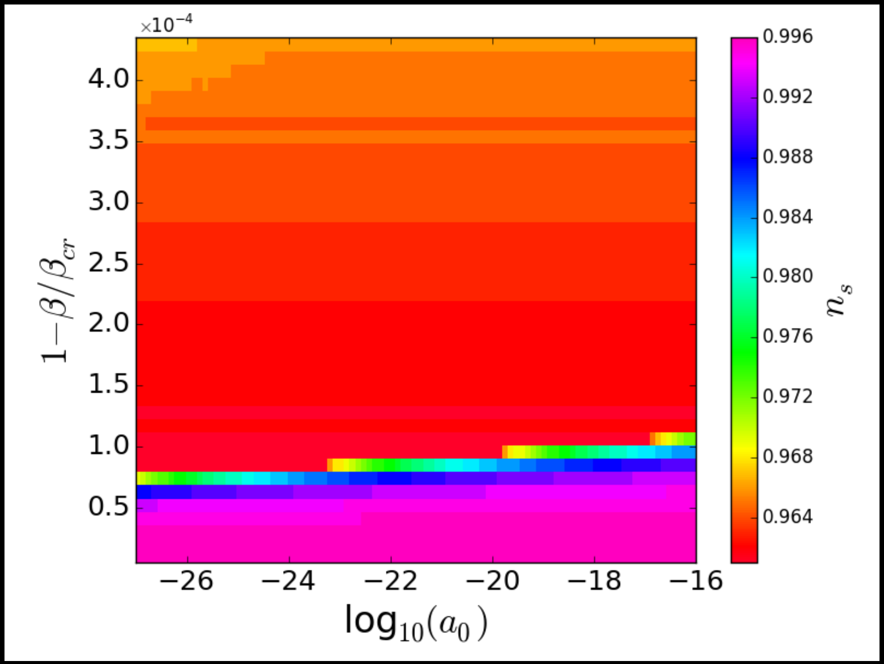


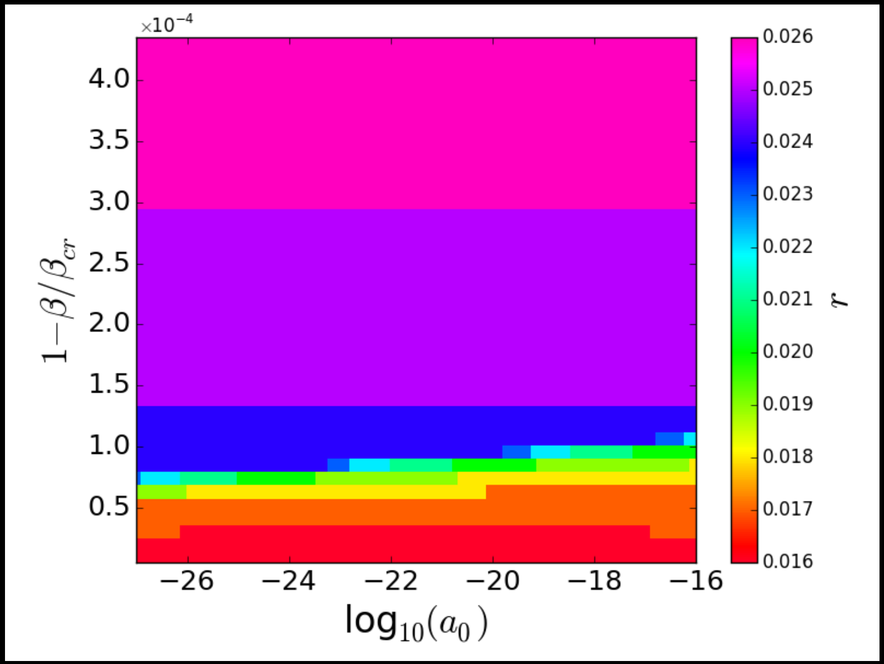
$\beta/\beta_{cr} = 0.9998$

Plateau-like potential – favored by Planck 2013.
Scalar-field plateau models of inflation: initial conditions problem, eternal inflation, unlikelihood (compared to power-law), several parameters: Ijjas, Steinhardt & Loeb, PLB 723, 261 (2013).

Torsion cosmology avoids these problems with only 1 parameter.

From the equivalent scalar field potential, one can calculate the parameters which are being measured in CMB.





Consistent with Planck 2015:

$n_s = 0.968 \pm 0.006$, $r < 0.12$

Not too sensitive to the initial scale factor.

Every black hole creates a new universe?

The closed Universe may have originated from the interior of a black hole existing in a parent universe when $C > (8/9)^{1/2}$.

Accordingly, every black hole may create a new, closed, baby universe (Novikov, Pathria, Hawking, Smolin, NP).

This hypothesis should solve the black hole information paradox: the information goes through the Einstein-Rosen bridge to the baby universe on the other side of the black hole's event horizon.

The motion through an event horizon is one way only: it defines the past and future. Time asymmetry at the event horizon may induce time asymmetry everywhere in the baby universe and explain why time flows in one direction.

Primordial fluctuations

The closed, homogeneous, and isotropic Universe can be viewed as a mechanical system described by a Lagrangian with the scale factor as a generalized coordinate – minisuperspace approximation.

$$L = \frac{6\pi^2}{\kappa}\left(\frac{-a\dot{a}^2}{c^2} + ka - \frac{1}{3}\kappa\epsilon a^3\right)$$

The Lagrange equations for the scale factor give the second Friedmann equation.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{a}} = \frac{\partial L}{\partial a}$$

Symmetry in time – conservation of energy.
The energy of the Universe = 0 gives the first Friedmann equation.

$$E = \frac{\partial L}{\partial \dot{a}}\dot{a} - L$$

Primordial fluctuations

The generalized momentum of the Universe.

$$p_a = \frac{\partial L}{\partial \dot{a}} = -\frac{12\pi^2 a\dot{a}}{\kappa c^2}$$

The uncertainty principle for the scale factor.

$$\Delta a \, \Delta \dot{a} \geq \frac{\hbar \kappa c^2}{24\pi^2 a}$$

Idea: quantum fluctuations of the scale factor, corresponding to quantum fluctuations of matter, may be the primordial fluctuations. Inflation, driven by torsion and particle production, increases them to macroscopic scales.

Cosmology without scalar fields. Work in progress.
NP, MPLA 33, 1850236 (2018)

Torsion as a solution to other problems

Torsion could also:

- Explain matter-antimatter asymmetry. The Dirac equation in presence of torsion is cubic in spinor fields. The cubic terms for matter and for antimatter have different signs relative to the mass term. This asymmetry is significant only when torsion is strong (early Universe). For an electron, $< 10^{-27}$ m.

- Explain a cosmological constant. If the metric tensor is proportional to the square of the torsion tensor, the field equations are the Einstein equations with the proportionality constant becoming a cosmological constant.

- Regularize Feynman diagrams in quantum field theory. In presence of torsion, translations do not commute, so the momentum operator components do not commute. Integration in momentum space must be replaced with summation over discrete momentum eigenvalues. Separation between adjacent eigenvalues increases with momentum. A sum is convergent even if the original integral was UV divergent.

NP, PLB 690, 73 (2010); PRD 83, 084033 (2011); GRG 46, 1625 (2014); arXiv:1712.09997

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References:

Kibble, JMP 2, 212 (1961)
Sciama, RMP 36, 463 (1964)
Hehl, von der Heyde & Kerlick, PRD 10, 1066 (1974)
Ellis & Madsen, CQG 8, 667 (1991)
Nomura, Shirafuji & Hayashi, PTP 86, 1239 (1991)
NP, arXiv:0911.0334; PLB 690, 73 (2010)
NP, PLB 687, 110 (2010); PLB 694, 181 (2010); GRG 44, 1007 (2012); PRD 85, 107502 (2012); Cubero & NP, CQG 37, 025011 (2020)
ApJ 832, 96 (2016); IJMPD 27, 1847020 (2018); MPLA 33, 1850236 (2018)
Desai & NP, PLB 755, 183 (2016); Unger & NP, ApJ 870, 78 (2019)

Summary

- The conservation law for total angular momentum (orbital + spin) in curved spacetime, consistent with Dirac equation, requires torsion.
- The simplest theory with torsion, Einstein-Cartan gravity, has the same Lagrangian as GR, but the affine connection contains the torsion tensor.
- Torsion is strong only at extremely high densities and manifests itself as gravitational repulsion that may avoid the formation of singularities. The Big Bang is replaced by a nonsingular Big Bounce.
- Particle production after a bounce can generate a finite period of inflation which ends when torsion becomes weak. No hypothetical fields/particles or extra dimensions are needed. The dynamics is plateau-like and supported by the Planck data.
- EC gravity appears to be the simplest and most natural explanation of the Big Bounce and inflation.
- Future work: explore how the presence of anisotropies affect the avoidance of singularities, and analyze the origin of the primordial fluctuations.